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HEATING OF A LIQUID-SATURATED POROUS MEDIUM BY AN ACOUSTIC FIELD

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The function of a volume heat source for the process of heating of a porous medium by an acoustic field has been constructed. The dependences of the power of the heat source on the parameters of the wave field and the parameters determining the state of the porous medium have been analyzed.

Environmental safety and the possibility of fairly simple technical solutions make the thermal methods of treatment of near-well zones of rocks using acoustic waves attractive. The basic mechanism converting the energy of the wave field in a porous medium to heat is the force of viscous friction between the saturating liquid and the skeleton of the porous medium.

Dynamics of the Temperature Field in a Homogeneous Porous Medium. Let the source of harmonic pressure waves be acting at the boundary ($x = 0$) of a liquid-saturated porous medium. In describing the wave and temperature problems in the system, we will assume that the temperature of the liquid and the skeleton of the porous medium coincide at each point and that the porous skeleton is incompressible. The latter assumption implies that the thermal effect for "fast" waves which propagate over the skeleton can be disregarded because of their weak attenuation. We assume that the nonuniformity of the temperature field exerts no influence on the acoustic pressure field (we disregard the influence of the temperature effects which are determined by viscosity and compressibility on the acoustic characteristics).

With allowance for the assumptions made for unsteady flow of the liquid in the porous medium we write the system of linearized momentum equations and equation of state:

$$m \frac{\partial \rho_{\text{liq}}}{\partial t} + \rho_{\text{liq}0} \frac{\partial u}{\partial x} = 0\,, \quad \rho_{\text{liq}0} \frac{\partial u}{\partial t} = - m \frac{\partial p}{\partial x} - \frac{m\mu}{k_*} u\,, \quad p = C_{\text{liq}}^2 \rho_{\text{liq}}\,, \quad x > 0\,. \tag{1}$$

The presence of the source of harmonic pressure waves at the boundary $x = 0$ can be written in the form of the following boundary condition:

$$p = A_p \cos \omega t\,, \quad x = 0\,, \quad t > 0\,. \tag{2}$$

We consider three cases for the right-hand boundary: 1) the porous medium is semiinfinite ($0 < x < \infty$), i.e., its length is much larger than the characteristic penetration depth of the acoustic waves; 2) the porous medium has a finite width ($0 < x < l$) and the boundary at $x = l$ is impermeable; 3) the boundary at $x = l$ is highly permeable. For actual situations the last condition implies, for example, that the near-well zone with a width l is "clogged" (region $0 < x < l$), and this zone is followed by an "unclogged" region with a permeability many times higher than the permeability in the near-well zone. For these cases the boundary conditions can be written as

$$u = 0 \quad (p = 0)\,, \quad x \to \infty\,; \tag{3a}$$

$$u = 0\,, \quad x = l\,; \tag{3b}$$

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$$p = 0\,, \quad x = l\,. \tag{3c}$$

Based on the system of equations (1) we can easily obtain

$$C_{\rm liq}^2 \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2} + \frac{1}{t_\mu}\frac{\partial p}{\partial t}\,, \quad t_\mu = \frac{k_* \rho_{\rm liq0}}{m\mu_{\rm liq}}\,. \tag{4}$$

The solution of (4) will be sought in the form

$$p\,(x, t) = C_1 \exp\,[-\,i\,(\omega t - Kx)] + C_2 \exp\,[-\,i\,(\omega t + Kx)]\,. \tag{5}$$

Here the first term characterizes the propagation of the wave from the source in the direction of the coordinate x while the second term characterizes its propagation in the opposite direction. With account for conditions (2) and (3) we obtain

$$p\,(x, t) = A_p \exp\,[-\,i\,(\omega t - Kx)]\,, \quad 0 \le x\,; \tag{6a}$$

$$p\,(x, t) = \frac{A_p}{1 + \exp\,(2iKl)}\,[\exp\,[-\,i\,(\omega t - Kx)] + \exp\,[-\,i\,(\omega t - K\,(2l - x))]]\,, \quad 0 \le x \le l\,; \tag{6b}$$

$$p\,(x, t) = \frac{A_p}{1 - \exp\,(2iKl)}\,[\exp\,[-\,i\,(\omega t - Kx)] - \exp\,[-\,i\,(\omega t - K\,(2l - x))]]\,, \quad 0 \le x \le l\,, \tag{6c}$$

$$K = \frac{\omega}{C_{\rm liq}}\sqrt{1 + \frac{i}{\omega t_\mu}}\,, \quad i = \sqrt{-1}\,.$$

The complex wave number K can be written in the form

$$K = k + i\delta\,, \quad k = \frac{\omega}{C_{\rm liq}\sqrt{2}}\sqrt{\sqrt{1 + (t_\mu \omega)^{-2}} + 1}\,, \quad \delta = \frac{\omega}{C_{\rm liq}\sqrt{2}}\sqrt{\sqrt{1 + (t_\mu \omega)^{-2}} - 1}\,. \tag{7}$$

The parameter δ shows the intensity of attenuation of the harmonic waves ($1/\delta$ is the characteristic depth of penetration of the pressure waves into the porous medium). In the case where the inertial effects are insignificant ($\omega t_\mu << 1$ and the momentum equation can be taken in the form of Darcy's law), Eq. (7) yields

$$k = \delta = \frac{1}{C_{\rm liq}}\sqrt{\frac{\omega}{2t_\mu}}\,.$$

Analogously to (6) for the velocity field we obtain

$$u\,(x, t) = A_u \exp\,[-\,i\,(\omega t - Kx)]\,, \quad A_u = \frac{A_p m\omega}{\rho_{\rm liq0} C_{\rm liq}^2 K}\,, \tag{8a}$$

$$u\,(x, t) = A_u\,[\exp\,[-\,i\,(\omega t - Kx)] - \exp\,[-\,i\,(\omega t - K\,(2l - x))]]\,, \quad A_u = \frac{A_p m\omega}{\rho_{\rm liq0} C_{\rm liq}^2 K\,[1 + \exp\,(2iKl)]}\,, \tag{8b}$$

$$u\,(x, t) = A_u\,[\exp\,[-\,i\,(\omega t - Kx)] + \exp\,[-\,i\,(\omega t - K\,(2l - x))]]\,, \quad A_u = \frac{A_p m\omega}{\rho_{\rm liq0} C_{\rm liq}^2 K\,[1 - \exp\,(2iKl)]}\,. \tag{8c}$$

The liquid saturating the porous medium executes vibrational motion relative to the rigid skeleton under the action of the harmonic pressure waves. Due to the viscous forces between the liquid and the skeleton, the energy of the wave changes to heat. The intensity of heating q referred to a unit volume of the porous medium will be equal to the power of friction forces in relative motion of the phases (of the liquid relative to the skeleton); for the intensity we can write

$$q = \frac{\mu}{k_*} (\mathrm{Re}\,(u))^2 . \tag{9}$$

Here Re(u) denotes the real part of the complex value of u.

Since the characteristic time of heating in the processes of practical interest is much longer than the period of vibrations of acoustic waves ($t >> \tau = 2\pi/\omega$), the most important parameter is the average heat influx per unit volume in a unit time:

$$Q = \frac{1}{\tau} \int_0^\tau q dt . \tag{10}$$

Substituting (8) into (10), we accordingly obtain

$$Q\,(x) = \frac{\mu A_p^2 m^2}{2k_* \rho_{\mathrm{liq}0}^2 C_{\mathrm{liq}}^2} \left(\sqrt{1 + (\omega t_\mu)^{-2}} \right)^{-1} \exp\,(-\,2\delta x) , \tag{11a}$$

$$Q\,(x) = Q_0\,(\exp\,(-\,2\delta x) - 2\exp\,(-\,2\delta l)\cos\,[2k\,(l - x)] + \exp\,[-\,2\delta\,(2l - x)]) , \tag{11b}$$

$$Q_0 = \frac{\mu A_p^2 m^2}{2k_* \rho_{\mathrm{liq}0}^2 C_{\mathrm{liq}}^2} \left(\sqrt{1 + (\omega t_\mu)^{-2}}\,(1 + 2\exp\,(-\,2\delta l)\cos\,(2kl) + \exp\,(-\,4\delta l)) \right)^{-1} ,$$

$$Q\,(x) = Q_0\,(\exp\,(-\,2\delta x) + 2\exp\,(-\,2\delta l)\cos\,[2k\,(l - x)] + \exp\,[-\,2\delta\,(2l - x)]) , \tag{11c}$$

$$Q_0 = \frac{\mu A_p^2 m^2}{2k_* \rho_{\mathrm{liq}0}^2 C_{\mathrm{liq}}^2} \left(\sqrt{1 + (\omega t_\mu)^{-2}}\,(1 - 2\exp\,(-\,2\delta l)\cos\,(2kl) + \exp\,(-\,4\delta l)) \right)^{-1} .$$

The first and third terms in Eqs. (11b) and (11c) characterize the heat influx due to the dissipation of the energy of the incident wave and the wave reflected from the surface $x = l$, while the second term determines the thermal effect caused by the interference of these waves. As the value of x increases, the first term decreases but the last term increases and the dependence of the interference term on the coordinate is oscillating.

The power of the acoustic radiator N is determined from the formula [1]

$$N = \frac{1}{\tau} \int_0^\tau \mathrm{Re}\,(p)\,\mathrm{Re}\,(u)\,dt . \tag{12}$$

Substituting (6) and (8) into formula (12), we obtain

$$N = \frac{A_p^2 m}{2\rho_{\mathrm{liq}0}\omega} \frac{k}{\sqrt{1 + (\omega t_\mu)^{-2}}} , \tag{13a}$$

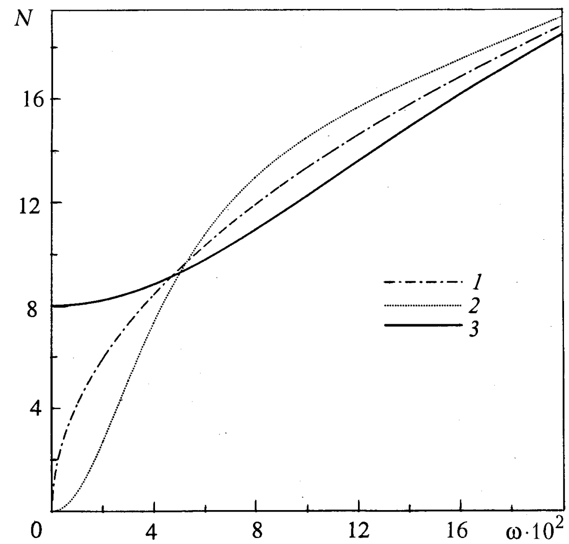

Fig. 1. Power of the acoustic field vs. frequency: 1) boundary condition (3a), 2) (3b), and 3) (3c). N, kW/m^2; ω, sec^{-1}.

$$N = \frac{A_p^2 m}{2\rho_{\text{liq}0}\omega} \frac{k - k\exp(-4\delta l) - 2\delta\exp(-2\delta l)\sin(2kl)}{\sqrt{1 + (\omega t_\mu)^{-2}}\,[1 + 2\exp(-2\delta l)\cos(2kl) + \exp(-4\delta l)]}, \tag{13b}$$

$$N = \frac{A_p^2 m}{2\rho_{\text{liq}0}\omega} \frac{k - k\exp(-4\delta l) + 2\delta\exp(-2\delta l)\sin(2kl)}{\sqrt{1 + (\omega t_\mu)^{-2}}\,[1 - 2\exp(-2\delta l)\cos(2kl) + \exp(-4\delta l)]}. \tag{13c}$$

Figure 1 gives the dependence of the acoustic-field power N on the frequency ω calculated from formulas (13) for the following values of the parameters of the acoustic field and the liquid-saturated porous medium: A_p = 2 MPa, m = 0.2, k_* = 10^{-12} m^2, μ = 0.001 Pa·sec, C_{liq} = 1500 m/sec, $\rho_{\text{liq}0}$ = 1000 kg/m^3, and l = 0.25 m. Curve 1 corresponds to the semiinfinite porous medium ($l = \infty$), curve 2 shows the porous medium at $x = l$ bordering an impermeable medium, and curve 3 corresponds to the porous medium at $x = l$ bordering a highly permeable medium. From the figure it is clear that for higher values of the frequency the power reaches the same asymptote for all three boundary conditions. When $\omega \to 0$, the power tends to zero for boundary conditions (3a) and (3b), though in different manners. For the boundary condition of the form (3c) we obtain

$$\lim_{\omega \to 0} N = \frac{1}{2} \frac{A_p^2 k_*}{l\mu}.$$

The equation of the influx of heat to the porous medium saturated with liquid, with allowance for the heat source due to the viscous attenuation of the acoustic field, will be written in the form

$$\rho c \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} + \tilde{Q}, \tag{14}$$

$$\rho c = (1 - m)\rho_s c_s + m\rho_{\text{liq}} c_{\text{liq}}, \; \lambda = \lambda_s (1 - m) + \lambda_{\text{liq}} m, \; \tilde{Q}(x) = Q(x),$$

$$0 \le x \le l; \; \tilde{Q}(x) = 0, \; l < x < \infty.$$

We take the initial condition for the temperature in the form

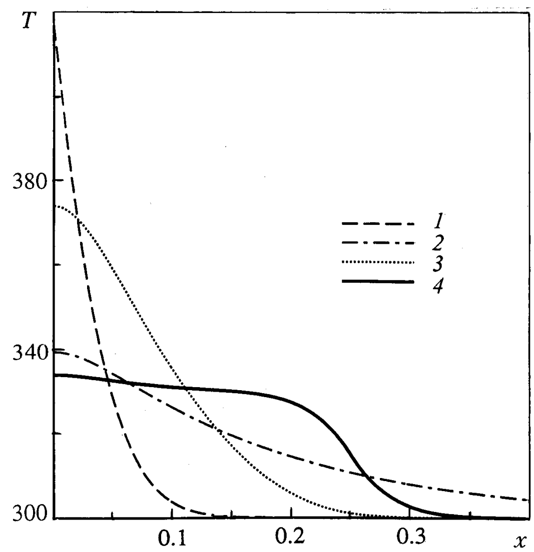

Fig. 2. Temperature distribution in the porous medium at the instant of time t = 2 h: 1) the case of a plane heat source; 2) boundary condition (3a); 3) (3b); 4) (3c). T, K; x, m.

$$T = T_0 \,, \quad x > 0 \,, \quad t = 0 \,. \tag{15}$$

We will assume that the boundary $x = 0$ is heat-insulated while at the boundary $x = l$ we have heat exchange with the environment. The temperature and the heat flux here are continuous. Then

$$\frac{\partial T}{\partial x} = 0 \,, \quad x = 0 \,; \quad [T] = 0 \,, \quad \left[\lambda \frac{\partial T}{\partial x}\right] = 0 \,, \quad x = l \,. \tag{16}$$

Here the square brackets denote an abrupt change in the value of the expression within them.

Based on the above equations we have performed calculations with the aim of analyzing the distinctive features of heating of the liquid-saturated porous medium depending on the state of the system and on the characteristics of the acoustic field. Figure 2 gives results of the temperature distribution at the instant of time t = 2 h under the action of an acoustic disturbance with a wave frequency of $\omega = 2 \cdot 10^2$ sec^{-1} and a source power of N = 2750 W/m^2 at the boundary of the porous medium. The value of the amplitude of the wave is equal respectively to A_p = 1.36 MPa for boundary condition (3a), A_p = 2 MPa for (3b), and A_p = 1.16 MPa for (3c). For the parameters determining the state and the geometric dimensions of the saturated porous medium we have taken the following values: ρ_s = 1400 kg/m^3, c_{liq} = 4200 J/(kg·K), c_s = 1520 J/(kg·K), λ_{liq} = 0.6 W/(m·K), λ_s = 0.5 W/(m·K), l = 0.25 m, and T_0 = 300 K. Curve 2 corresponds to the semiinfinite porous medium ($l = \infty$), curve 3 shows the porous medium at $x = l$ bordering an impermeable medium, and curve 4 corresponds to the porous medium bordering a highly permeable medium. From Fig. 2 it is clear that the effect of heating in the zone $0 \le x \le l$ is largest in the case where the right-hand border is highly permeable. Here the porous medium is heated more uniformly and the average level of temperature is higher than that in the remaining two cases. Thus, when the porous medium borders a higher-permeability medium, by selecting the frequency of acoustic waves depending on the thickness l of a low-permeability zone one can achieve a more uniform distribution of the heat-source power in the depth of the porous medium.

For the sake of comparison we considered the problem of heating of a porous medium by a plane heat source. Let a heat source with a constant flux equal to the power of the acoustic field N be located at the boundary $x = 0$ of the porous medium:

$$\lambda \frac{\partial T}{\partial x} = -N \,, \quad x = 0 \,, \; t > 0 \,. \tag{17}$$

In this case the heat is propagating in it only by heat conduction. The initial condition for the temperature is determined from (15), while the heat-conduction equation is determined from (14). In (14), we have $\tilde{Q}(x) = 0$ for all $0 \le x$ and $0 \le t$. The solution of such a problem has the following form [2]:

$$T - T_0 = \frac{N}{\lambda} \left\{ 2 \sqrt{\frac{\chi t}{\pi}} \exp\left(-\frac{x^2}{4\chi t}\right) - x \operatorname{erfc}\left(\sqrt{\frac{x^2}{4\chi t}}\right) \right\} , \quad \chi = \frac{\lambda}{\rho c} . \tag{18}$$

Curve 1 in Fig. 2 corresponds to the case of a plane heat source. As is clear from the figure, the main portion of supplied heat is localized here in the boundary zone. Thus, by exposure to an acoustic field with a rather moderate intensity one can achieve a much deeper heating of the system over the same period of time than in the case where the thermal action is carried out only by heat conduction.

Dynamics of the Temperature Field in an Inhomogeneous Porous Medium. Let us consider a two-zone porous medium with porosity m_1 and permeability $k_{*(1)}$ in the first zone $(0 < x \le x_s)$ and with m_2 and $k_{*(2)}$ in the second zone $(x_s < x \le l)$ at whose boundary $(x = 0)$ a source of harmonic pressure waves is acting. We will assume that the assumptions made for the case of a homogeneous porous medium are also true here.

For the two-zone medium the system of linearized equations of continuity, momentum, and state has the form

$$m_j \frac{\partial \rho_{\text{liq}}}{\partial t} + \rho_{\text{liq0}} \frac{\partial u}{\partial x} = 0 , \quad \rho_{\text{liq0}} \frac{\partial u}{\partial t} = - m_j \frac{\partial p}{\partial x} - \frac{m_j \mu}{k_{*(j)}} u , \quad p = C_{\text{liq}}^2 \rho_{\text{liq}} , \tag{19}$$

where $j = 1$ for $0 < x \le x_s$, $j = 2$ for $x_s < x \le l$, and $j = 3$ for $l < x < \infty$.

The boundary condition at $x = 0$ is determined from formula (2), and for the boundary $x = x_s$ we write the conditions of absence of an abrupt change in the pressure and the velocity of motion of the liquid:

$$[p] = 0 , \quad [u] = 0 , \quad x = x_s . \tag{20}$$

We consider the following cases for the right-hand boundary of the second zone:
it is impermeable

$$u = 0 , \quad x = l \tag{21a}$$

or highly permeable

$$p = 0 , \quad x = l . \tag{21b}$$

We find the velocity field for the system of equations (19). For the first zone $0 < x \le x_s$ with boundary conditions (21a) and (21b) we obtain

$$u(x, t) = \frac{A_p m_2 \omega}{\rho_{\text{liq0}} C_{\text{liq}}^2 K_2} [C_2 \cosh(iK_1 x) - \exp(-iK_1 x)] \exp(-i\omega t) ,$$

$$K_j = \frac{\omega}{C_{\text{liq}}} \sqrt{1 + \frac{i}{\omega t_{\mu(j)}}} , \quad t_{\mu(j)} = \frac{k_{*(j)} \rho_{\text{liq0}}}{m_j \mu_{\text{liq}}} . \tag{22}$$

For the second zone $x_s < x \le l$ with the same boundary conditions we accordingly have

$$u(x, t) = - \frac{A_p m_2 \omega}{\rho_{\text{liq0}} C_{\text{liq}}^2 K_2} \frac{C_2 \sinh(iK_1 x_s) + \exp(-iK_1 x_s)}{\cosh[iK_2 (l - x_s)]} \sinh[iK_2 (l - x)] \exp(-i\omega t) , \tag{23a}$$

$$C_2 = \exp(-iK_1 x_s) \frac{m_2 K_1 \sinh[iK_2 (x_s - l)] + m_1 K_2 \cosh[iK_2 (x_s - l)]}{m_1 K_2 \cosh(iK_1 x_s) \cosh[iK_2 (x_s - l)] - m_2 K_1 \sinh(iK_1 x_s) \sinh[iK_2 (x_s - l)]} ,$$

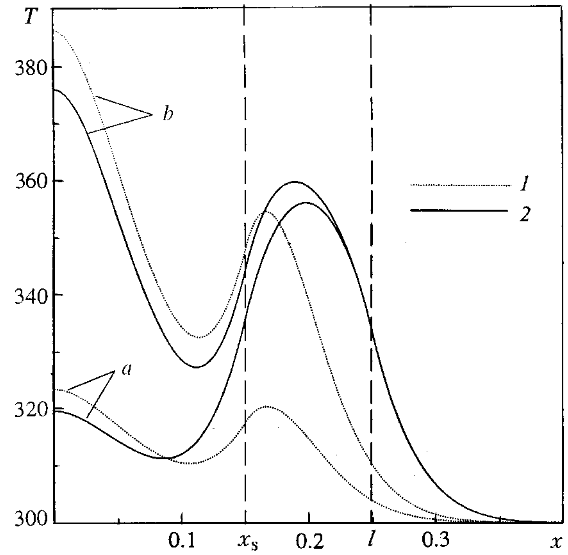

Fig. 3. Temperature distribution in an inhomogeneous porous medium at the instant of time t = 2 h: 1) boundary condition (21a); 2) (21b) [a) $\omega = 5{\cdot}10^2$ $\sec^{-1}$; b) 10^3 $\sec^{-1}$]. T, K; x, m.

$$u(x, t) = -\frac{A_p m_2 \omega}{\rho_{\text{liq}0} C_{\text{liq}}^2 K_2} \frac{C_2 \sinh(iK_1 x_s) + \exp(-iK_1 x_s)}{\sinh[iK_2(l - x_s)]} \cosh[iK_2(l - x)] \exp(-i\omega t), \tag{23b}$$

$$C_2 = \exp(-iK_1 x_s) \frac{m_2 K_1 \cosh[iK_2(x_s - l)] + m_1 K_2 \sinh[iK_2(x_s - l)]}{m_1 K_2 \cosh(iK_1 x_s) \sinh[iK_2(x_s - l)] - m_2 K_1 \sinh(iK_1 x_s) \cosh[iK_2(x_s - l)]}.$$

We write the heat-conduction equation for the two-zone porous medium:

$$\rho_j c_j \frac{\partial T}{\partial t} = \lambda_j \frac{\partial^2 T}{\partial x^2} + \tilde{Q},$$

$$\rho_j c_j = (1 - m_j) \rho_s c_s + m_j \rho_{\text{liq}} c_{\text{liq}}, \quad \lambda_j = \lambda_s (1 - m_j) + \lambda_{\text{liq}} m_j,$$

$$\tilde{Q}(x) = Q(x), \quad 0 < x \le l; \quad \tilde{Q}(x) = 0, \quad l < x < \infty.$$

The volume heat influx $Q(x)$ for each zone is determined using (9), (10), (22), and (23).

We will take the initial and boundary conditions for the temperature analogous to the case of a homogeneous porous medium and determine them from (15) and (16). The temperature and the heat flux are continuous at the boundaries $x = x_s$ and $x = l$:

$$[T] = 0, \quad \left[\lambda_j \frac{\partial T}{\partial x}\right] = 0, \quad x = x_s, \quad x = l.$$

Figure 3 shows the temperature distributions at the instant of time t = 2 h in the case of exposure to an acoustic disturbance with parameters of the wave of A_p = 2 MPa and $\omega = 5{\cdot}10^2$ $\sec^{-1}$ (curves a) and $\omega = 10^3$ $\sec^{-1}$ (curves b) at the boundary of the inhomogeneous porous medium. The parameters of the porous medium have the same values as in the first problem, except for $m_1 = m$, $k_* = 10^{-10}$ m^2, $m_2 = m(1 - \nu)$, $m = 0.2$, and $\nu = 0.75$. The value of the permeability for each zone was determined using the Kozeny formula [3]:

$$k_{*(j)} = k_* \frac{m_j^3}{(1 - m_j)^2} .$$

From the figure it is clear that an increase in the frequency has no desirable effect, since the main portion of the expended energy goes to heat the near zone. We emphasize that the temperature maximum is observed inside the porous medium in the zone with a lower permeability and not at the boundary (curves b). This suggests that we can achieve efficient action on clogged zones by supplying an acoustic field.

The results obtained show that, by selecting the frequency and amplitude of the waves depending on the parameters of the porous medium and the liquid saturating it and also on the boundary conditions, one can achieve a more efficient action of the acoustic field on the near-face zone.

NOTATION

p and ρ_{liq}, disturbance of the pressure and the density in the liquid, Pa and kg/m^3; ρ_{liq0}, liquid density corresponding to the undisturbed state, kg/m^3; u, filtration rate, m/sec; m, porosity; k_*, permeability, m^2; μ, liquid viscosity, Pa·sec; C_{liq}, velocity of sound in the saturating liquid, m/sec; t, time, sec; x, coordinate, m; A_p and ω, amplitude and angular frequency of the wave, Pa and 1/sec; K, complex wave number; δ and k, parameters; i, imaginary unit; q, intensity of heating, W/m^3; τ, period of vibrations, 1/sec; Q, average heat influx, W/m^3; N, power of the acoustic field, W/m^2; T and λ, temperature and thermal conductivity of the liquid-saturated skeleton of the porous medium, K and W/(m·K); c_{liq}, heat capacity, J/(kg·K); ρ_s and c_s, density and heat capacity of the porous medium, kg/m^3 and J/(kg·K); λ_{liq} and λ_s, thermal conductivity of the liquid and the skeleton of the porous medium, W/(m·K); K_j, wave number for the jth zone; m_j and $k_{*(j)}$, porosity and permeability of the porous medium in the jth zone, m^2; ν, degree of clogging of the porous medium. Subscripts: j = 1, 2, 3, numbers of the zone; liq, liquid; s, porous medium; 0, initial.

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